# Determination of the Absolute Propagation Rate Constants in Polymerization with Reversible Aggregation of Active Centers<sup>†</sup>

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ABSTRACT: When the propagating active species  $(P_n^*)$  aggregate reversibly into the nonreactive (dormant) species,  $mP_n^* \rightleftharpoons (P_n^*)_m$ ,  $P_n^* + \mathbf{M} \rightharpoonup P_{n+1}^*$ , both the absolute rate constant of propagation  $(k_p)$  and the equilibrium constant of aggregation  $(K_a)$  can be determined simultaneously by using an equation derived recently, assuming polymerization to be living:  $r_p^{1-m} = -mK_a/k_p^{m-1} + k_p[\mathbf{I}]_0r_p^{-m}$  (where  $r_p = -\mathbf{d} \ln[\mathbf{M}]/\mathrm{d}t$ , m is the aggregation degree,  $[\mathbf{M}]$  denotes the instantaneous concentration of monomer, and  $[\mathbf{I}]_0$  is the starting concentration of initiator  $([\mathbf{I}]_0 = [P_n^*] + m[(P_n^*)_m])$ ). This equation has been applied in the present work to analyze several polymerizations studied by other authors who previously used numerical methods in determining the involved rate and equilibrium constants. A few systems have been chosen, namely, the anionic polymerization with the Li<sup>+</sup> counterion of o-methoxystyrene, methyl methacrylate, and hexamethylcyclotrisiloxane, as well as the anionic polymerization of oxirane with the Cs<sup>+</sup> counterion. The  $k_p$  and  $K_a$  found analytically are in good agreement with those determined previously numerically.

### Introduction

Aggregation of active centers in the anionic polymerization of vinyl, diene,  $^{1-7}$  acrylic,  $^{8,9}$  and heterocyclic monomers  $^{10-16}$  is a well-documented phenomenon. Recently, we have found that in the polymerization of  $\epsilon$ -caprolactone initiated with dialkylaluminum alkoxides (R<sub>2</sub>AlOR'), the resulting macroalkoxide growing species R'O[C(O)(CH<sub>2</sub>)<sub>5</sub>O]<sub>n</sub>C(O)(CH<sub>2</sub>)<sub>4</sub>CH<sub>2</sub>O-AlR<sub>2</sub> also undergo aggregation with the aggregation degree depending on the substituent on the aluminum atom, additional complexing agents, and the solvent used.  $^{17-19}$ 

Usually, the reactivity of the aggregated species differs substantially from that of the nonaggregated ones. Therefore aggregation of the active centers affects the kinetics of polymerization. The nonaggregated species, most often, are the more reactive, as suggested by the fractional orders (externally) of propagation with respect to the active centers. 1–16

The aggregation of the initiator may also have important consequences. Our recent results on the polymerization of  $\epsilon$ -caprolactone with Al[OCH(CH<sub>3</sub>)<sub>2</sub>]<sub>3</sub><sup>19</sup> indicate that the aggregates of a given initiator with various aggregation degrees may substantially differ in reactivity: for example,  $\{Al[OCH(CH_3)_2]_3\}_3$  leads to a much more rapid polymerization of  $\epsilon$ -caprolactone than that initiated by  $\{Al[OCH(CH_3)_2]_3\}_4$ .

In the majority of polymerizations studied till now and involving aggregates of active centers, propagation is first order internally with respect to monomer.<sup>1-14,16-19</sup> Thus, the pertinent kinetic scheme for propagation proceeding exclusively on the nonaggregated species reads

$$(\mathbf{P}_n^*)_m \stackrel{K_{da}}{\rightleftharpoons} m \mathbf{P}_n^* \qquad (K_{da} = 1/K_a)$$
 (1)

$$\mathbf{P}_n^* + \mathbf{M} \xrightarrow{k_p} \mathbf{P}_{n+1}^* \tag{2}$$

where

$$[I]_0 = [P_n^*] + m[(P_n^*)_m]$$
 (3)

 $\mathbf{P}_n^*$  and  $(\mathbf{P}_n^*)_m$  denote nonaggregated and aggregated active centers, respectively, m is the aggregation degree,  $\mathbf{M}$  is the monomer,  $K_{\mathrm{da}}$  and  $K_{\mathrm{a}}$  are the equilibrium constants of deaggregation and aggregation, respectively,  $k_{\mathrm{p}}$  is the propagation rate constant of the nonaggregated species, and  $[\mathbf{I}]_0$  is the starting concentration of initiator.

The kinetics of the monomer consumption is described by the following equations:

$$r_{\rm p} = -\frac{{
m d}[{\bf M}]}{[{\bf M}] {
m d}t} = -\frac{{
m d} \ln[{\bf M}]}{{
m d}t} = k_{\rm p}[{\bf P}_n^*]$$
 (4)

or, in the integrated form,

$$r_{\rm p} = {\ln([{\bf M}]_0/[{\bf M}])}/t = k_{\rm p}[{\bf P}_n^*]$$
 (5)

Thus

$$\ln([\mathbf{M}]_0/[\mathbf{M}]) = k_n[\mathbf{P}_n^*]t \tag{6}$$

Results of the kinetic measurements are presented in the form of plots of  $\ln([\mathbf{M}]_0/[\mathbf{M}])$  vs polymerization time (t), and  $r_p$  is given by the slope. The rate constant of propagation cannot, however, be determined directly from the kinetic measurement, because the concentration of the actually growing, nonaggregated species  $[\mathbf{P}_n^*]$  is not known (cf. eq 5).

It is also usually assumed in the analysis of systems with aggregation of  $\mathbf{P}_n^*$  that active centers are present predominantly in the aggregated form; i.e., equilibrium 1 is shifted to the left-hand side,  $[\mathbf{P}_n^*] \approx 0$ , and practically  $[\mathbf{I}]_0 = m[(\mathbf{P}_n^*)_m]$ . Finally, the correlation between the relative polymerization rate  $(r_p)$  and the total concentration of the active centers  $([\mathbf{I}]_0)$  is obtained (with the above given assumption:  $m[(\mathbf{P}_n^*)_m] \gg [\mathbf{P}_n^*]$ ):

$$\log r_{\rm p} = \log\{k_{\rm p}(K_{\rm da}/m)^{1/m}\} + (1/m)\log[{\bf I}]_0 \eqno(7)$$

Then, plotting  $\log r_{\rm p}$  vs  $\log[{\rm I}]_0$ , one obtains the aggregation degree m directly from the slope of the resulting straight line. The intercept gives the product  $k_{\rm p} K_{\rm ds}^{1/m}$ .

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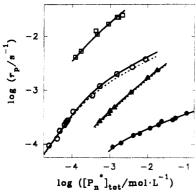


Figure 1. External orders in active centers: dependencies of  $\log r_p$  against  $\log [\mathbf{P}_n^*]_{\text{tot}}$ . ( $\Delta$ ) Polymerization of o-methoxystyrene on  $\cdot \cdot \cdot - CH_2CH[(o-CH_3O)C_6H_4], Li^+ ion pairs, toluene as a solvent,$ 20 °C (experimental data taken from ref 7). (□) Polymerization of methyl methacrylate on ···-CH<sub>2</sub>C(CH<sub>3</sub>)C(O)OCH<sub>3</sub>,Li<sup>+</sup> ion pairs, THF as a solvent, -65 °C (experimental data taken from ref 9). (O) Polymerization of oxirane on  $\cdot \cdot \cdot - CH_2CH_2O^-$ , Cs<sup>+</sup> ion pairs, THF as a solvent, 70 °C (experimental data taken from ref 10). ( $\bullet$ ) Polymerization of D<sub>3</sub> on  $\cdots$ -Si(CH<sub>3</sub>)<sub>2</sub>O-, Li<sup>+</sup> ion pairs, THF as a solvent, 22 °C (experimental data taken from ref 14). Points, experimental; lines, calculated by eq 12; solid lines,  $k_p$ and Ka determined by the straight-line correlation method (eq 8); dashed lines,  $k_p$  and  $K_a$  determined by the numerical treatment.

The absolute values of  $k_p$  and  $K_{da}$  are, however, inaccessible when this approach is used. On the other hand, several kinetic studies were reported giving plots of  $\log r_{\rm p}$  vs  $\log[I]_0$  with a marked curvature; 5-11,13,14,18 some of these are collected in Figure 1. This behavior may stem from the presence of the nonaggregated and aggregated species at comparable concentrations; hence the term  $[\mathbf{P}_n^*]$ in eq 3 cannot be neglected, and eq 7 cannot be used to analyze the polymerization kinetics. In fact, plots of log  $r_{\rm p}$  against  $\log[\mathbf{I}]_0$  should "always" be curved. The reasons that the curvature is not observed are related to the accuracy of the kinetic measurements and the range of the  $[I]_0$  applied.

Apparently, there were attempts to solve the kinetic scheme given by eqs 1 and 2 without any simplifying assumption like  $[P_n^*] \approx 0$ . However, it seems that the proper analytical solution has not been applied, because a number of authors analyzed kinetics described by eqs 1 and 2 numerically, determining  $k_p$  and  $K_a$  this way.

We have recently found a way of solving analytically the discussed kinetic scheme (propagation on the nonaggregated species, reversibly aggregating into the inactive species), giving direct access to both  $k_p$  and  $K_a$ . The success is based on operating with two experimentally measurable variables  $(r_p \text{ and } [I]_0)$  and two parameters  $(k_p \text{ and } K_a)$  in such a way that a straight-line relation is obtained. This relatively simple equation reads<sup>17</sup>

$$r_{\rm p}^{1-m} = -m/K_{\rm da}k_{\rm p}^{m-1} + k_{\rm p}[{\bf I}]_0 r_{\rm p}^{-m}$$
 (8)

where  $r_p = \{\ln([\mathbf{M}]_0/[\mathbf{M}])\}/t = k_p[\mathbf{P}_n^*]$ . Thus when  $r_p^{m-1}$  is plotted against  $[\mathbf{I}]_0r_p^{-m}$ , the rate constant  $k_p$  is obtained directly from the slope, and  $K_{\mathrm{da}}$  is calculated after determining the intercept. The  $r_p$  are measured from the slopes of the dependencies of  $ln([M]_0/[M])$  vs time. The value of m is either assumed (m is usually equal to 2, 3, or 4) or predetermined experimentally by using eq 7. The method described above has successfully been applied in our recent analysis of the polymerization of  $\epsilon$ -caprolactone initiated with dialkylaluminum alkoxides (R2AlOR').17,18

We should add that Szwarc correctly pointed out that there should be an analytical solution for the kinetic equations describing the set of reactions (1) and (2)3 and then gave the solution for m = 2 but did not analyze any kinetic data.5

In the present paper the analytical approach described above and called the "linear analytical method" is applied to a few cases treated previously numerically.

# Determination of $k_p$ and $K_{da}$ by a Linear Analytical Method

Anionic Polymerization of o-Methoxystyrene. van Beylen and Smets<sup>7</sup> studied the kinetics of the anionic polymerization of o-methoxystyrene with Li<sup>+</sup> counterion in toluene as a solvent at 20 °C. The order of propagation increased from 1/m = 0.51 to 0.67 when the total concentration of the active centers ( $[\mathbf{P}_n^*]_{tot} = [\mathbf{P}_n^*] +$  $m[(\mathbf{P}_n^*)_m]$  decreased from  $1.82 \times 10^{-2}$  to  $5.32 \times 10^{-4}$  mol·L<sup>-1</sup> (Figure 1, experimental points represented by triangles).  $[P_n^*]_{tot}$  were determined from molecular weights of the resulting poly(o-methoxystyrene). A slight curvature of the  $\log r_p \div \log[\mathbf{P}_n^*]_{\text{tot}}$  plot is clearly seen. The authors assumed that the actually growing species, lithium alkyl ion pairs, were in dynamic equilibrium with their dimers. The latter species were considered to be unreactive toward monomer. In this system, the concentration of the reactive, monomeric active centers was described by the following expression:7

$$[\mathbf{P}_n^*] = -K_{da}/4 + (K_{da}^2/4 + 2K_{da}[\mathbf{P}_n^*]_{tot})^{1/2}/2$$
 (9)

where  $K_{da}$  denotes the equilibrium constant of deaggregation,  $K_{da} = 1/K_a$ . In further treatment van Beylen and Smets assumed the values of  $K_{da}$  and calculated  $[\mathbf{P}_n^*]$  for various  $[\mathbf{P}_n^*]_{\text{tot}}$  from eq 9. Then the measured  $r_p$  were correlated graphically with the calculated  $[P_n^*]$ . The whole procedure was repeated several times until the best fit to the data was obtained for a certain  $K_{da}$ . Finally, from the relationship  $r_p = k_p[\mathbf{P}_n^*]$ , the propagation rate constant  $k_p$  was computed for various  $r_p$  and  $[\mathbf{P}_n^*]$ . The best fit to the experimental data was obtained for  $k_p = 0.83 \text{ mol}^{-1} \cdot \text{L·s}^{-1}$  and  $K_{\text{da}} = 10^{-3} \text{ mol} \cdot \text{L}^{-1}$  (i.e.,  $K_{\text{a}} = 10^3$  $mol^{-1}\cdot L$ ).

Application of our eq 8 to the same set of experimental data is illustrated in Figure 2a.  $k_p = 0.75 \text{ mol}^{-1} \cdot \text{L} \cdot \text{s}^{-1}$  and  $K_a = 0.86 \times 10^3 \,\mathrm{mol^{-1}\cdot L}$  were determined, close enough to the values obtained by Van Beylen and Smets. The respective plot of  $r_p^{-1}$  vs  $[\mathbf{P}_n^*]_{tot}r_p^{-2}$ , shown in Figure 2a, gives a good straight line with a linear regression coefficient (r) equal to 0.991.

Anionic Polymerization of Methyl Methacrylate. Müller et al.<sup>8,9</sup> studied the anionic polymerization of methacrylates with Li<sup>+</sup> as counterion in THF solvent at -65 °C. For methyl methacrylate (MMA) the slope of the tangents of the plot  $\log r_{\rm p}$  vs  $\log [{\bf P}_n^*]_{\rm tot}$  increased from 1/m = 0.58 to 0.78 when  $[{\bf P}_n^*]_{\rm tot}$  decreased from  $2.1 \times 10^{-3}$ to  $1.2 \times 10^{-4}$  mol·L<sup>-1</sup> (Figure 1, experimental points represented by squares). Under these polymerization conditions the presence of "free" ions was excluded and dimerization of the active centers was assumed. The molar fractions of the monomeric (nonaggregated) ion pairs  $(\alpha)$ were calculated from eq 10:

$$\alpha = \frac{[\mathbf{P}_n^*]}{[\mathbf{P}_n^*]_{\text{tot}}} = \frac{(1 + 8K_a[\mathbf{P}_n^*]_{\text{tot}})^{1/2} - 1}{4K_a[\mathbf{P}_n^*]_{\text{tot}}}$$
(10)

The monomeric ion pairs were assumed to be the only propagating active species. Finally,  $k_p = 46 \text{ mol}^{-1} \cdot \text{L} \cdot \text{s}^{-1}$ and  $K_a = 2 \times 10^{-3} \,\text{mol}^{-1} \cdot \text{L}$  were found numerically by using a nonlinear least squares method.

Application of the linear analytical method, based again on eq 8, gives a plot with  $k_p = 43.6 \text{ mol}^{-1} \cdot \text{L} \cdot \text{s}^{-1}$  and  $K_a =$ 

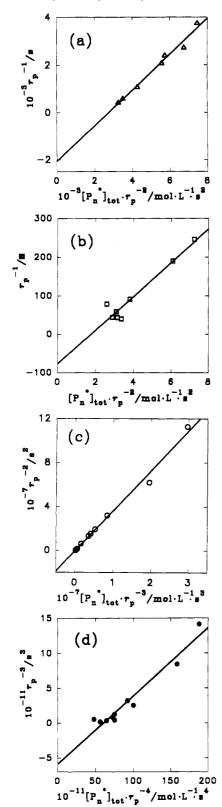


Figure 2. Plots of  $r_p^{1-m}$  vs  $[P_n^*]_{tot}r_p^{-m}$  according to eq 8: determination of the propagation rate constants  $(k_p)$  and the constants of aggregation equilibria  $(K_a)$ . Anionic polymerization of (a) o-methoxystyrene, (b) methyl methacrylate, (c) oxirane, and (d) hexamethylcyclotrisiloxane. Other conditions are as in the caption to Figure 1.

 $1.67 \times 10^3$  mol·L<sup>-1</sup> (Figure 2b), again close enough to the values determined previously by the numerical treatment.

Anionic Polymerization of Oxirane. Kazanskii et al. 10,11 studied the anionic polymerization of oxirane with Na<sup>+</sup>, K<sup>+</sup>, and Cs<sup>+</sup> as counterions in ethereal solvents. For all of these counterions fractional external orders in active centers at high concentrations were observed. For example, for Cs+ counterion in THF at 70 °C the order decreased from 1/m = 1 to 1/3 when the total concentration of active centers increased from  $2.5 \times 10^{-5}$  to  $9 \times 10^{-3}$  $\text{mol-L}^{-1}$ . The curvature of the plot  $\log r_{\text{p}}$  against  $\log [\mathbf{P}_{n}^{*}]_{\text{tot}}$ is illustrated in Figure 1 (experimental points represented by hollow circles). Conductometric studies excluded the presence of unpaired ions. Thus, apparently, cesium alkoxide chain ends aggregate into trimers. Kazanskii determined the absolute propagation rate constant directly from the kinetic data at low  $[P_n^*]_{tot}$  when m = 1. At these conditions  $k_p^{app} = k_p$  (where  $k_p^{app} = r_p/[\mathbf{P}_n^*]_{tot}$ ). At higher  $[\mathbf{P}_n^*]_{tot}$  the molar fraction of the monomeric ion pairs  $\alpha$ =  $k_{\rm p}^{\rm app}/k_{\rm p}$ . Thus, the aggregation equilibrium constant  $K_a$  could be calculated from eq 11 as

$$K_{\rm a} = \frac{1 - \alpha}{3\alpha^3 [\mathbf{P}_n^*]_{\rm tot}^2} \tag{11}$$

This approach gave finally  $k_p = 3.5 \text{ mol}^{-1} \cdot \text{L} \cdot \text{s}^{-1}$  and  $K_a =$  $3 \times 10^6 \text{ mol}^{-1} \cdot \text{L}$ .

Our treatment gave  $k_p = 3.56 \text{ mol}^{-1} \cdot \text{L} \cdot \text{s}^{-1}$  and  $K_a = 1.99$  $\times$  10<sup>6</sup> mol<sup>-1</sup>·L, as a result of plotting the kinetic data in the coordinates  $r_{\rm p}^{-2}$  vs  $[{\bf P}_n^*]_{\rm tot}r_{\rm p}^{-3}$  (Figure 2c).

Anionic Polymerization of Hexamethylcyclotri-

siloxane (D<sub>3</sub>). Wilczek and Kennedy<sup>14</sup> studied the kinetics of the anionic polymerization of D<sub>3</sub> with Li<sup>+</sup> counterion in THF as a solvent at temperatures from 0 to 60 °C. External orders in the active centers were fractional and depended on their total concentration. For example, at 22 °C the slope of the plot of  $\log r_{\rm p}$  vs  $\log [{\bf P_n}^*]_{\rm tot}$  increased from 0.25 to  $\sim$  0.33 when  $[{\bf P_n}^*]_{\rm tot}$  decreased from 0.11 to  $1.2 \times 10^{-3}$  mol·L<sup>-1</sup> (Figure 1, experimental points represented by filled circles). Under these conditions polymerization of D<sub>3</sub> proceeds exclusively with lithium silanolate ion pairs as the active centers. Consequently, it was assumed that the active centers aggregate into nonreactive tetramers and, possibly, trimers, being in a dynamic equilibrium with the actually propagating, nonaggregated species. Finally, the kinetic data were tentatively analyzed by means of eq 7, and therefore only an apparent rate constant of propagation was determined.

We have analyzed these kinetic data with eq 8 assuming first m = 4. The result of this analysis is given in Figure 2d: the plot of  $r_p^{-3}$  vs  $[\mathbf{P}_n^*]_{tot}r_p^{-4}$  gives a satisfactory straight-line correlation (r=0.972).  $k_p=0.098\,\mathrm{mol}^{-1}\cdot\mathrm{L}\cdot\mathrm{s}^{-1}$ and  $K_a = 1.37 \times 10^8 \,\mathrm{mol}^{-3} \cdot L^3$  were determined. It should be stressed that an attempt to use eq 8, but with the assumption that m = 3, for the analysis of these data was unsuccessful: the linear regression coefficient was as low as r = 0.22 and the slope of the resulting line was negative, leading to  $k_p < 0$ . This result suggests that the ···-Si(CH<sub>3</sub>)<sub>2</sub>O-,Li<sup>+</sup> ion pairs aggregate in the discussed system predominantly into tetramers.

# Calculation of the log $r_p$ vs $log[P_n^*]_{tot}$ Dependencies

Equation 7, relating the rate of propagation with the total concentration of the active centers, was derived after assuming  $[P_n^*] = 0$ . If  $[P_n^*]$  is not neglected, the respective equation contains an additional term in comparison with eq 7, namely 18

$$\log r_{\rm p} = \log\{k_{\rm p}(K_{\rm da}/m)^{1/m}\} + (1/m)\log[\mathbf{I}]_0 + \log \mathbf{X} \quad (12)$$

where the function X reads

$$\mathbf{X} = (K_{de}/8[\mathbf{I}]_0 + 1)^{1/2} - (K_{de}/8[\mathbf{I}]_0)^{1/2} \text{ (for } m = 2)$$
 (13)

Table 1. Anionic Polymerization with Reversible Aggregation of Active Centers: Comparison of Propagation Rate Constants  $(k_p)$  and Equilibrium Constants of Aggregation  $(K_a)$  Determined Numerically with Constants Determined by Means of Eq 8

active species	CH <sub>2</sub> CH[C <sub>6</sub> H <sub>4</sub> (OCH <sub>3</sub> )],Li <sup>+</sup>	···-CH <sub>2</sub> C(CH <sub>3</sub> )C(O)OCH <sub>3</sub> ,Li <sup>+</sup>	···-CH <sub>2</sub> CH <sub>2</sub> O-,Li+	Si(CH <sub>3</sub> ) <sub>2</sub> O-,Li <sup>+</sup>
monomer	o-methoxystyrene	methyl methacrylate	oxirane	$D_3$
solvent, temp	toluene, 20 °C	THF, -65 °C	THF, 70 °C	THF, 22 °C
ref	7	9	11	14
aggregation $deg(m)$	2	2	3	4
$k_{\rm p}/({\rm mol^{-1} \cdot L \cdot s^{-1}})$				
numerical curve fit	0.83	46.0	3.50	
present work (eq 8)	0.75	43.6	3.65	0.098
$K_{\mathbf{a}}/(\mathrm{mol}^{-1}\cdot\mathbf{L})^{m-1}$				
numerical curve fit	1000	2000	$3.00 \times 10^{6}$	
present work (eq 8)	857	1670	$1.99 \times 10^{6}$	$1.37 \times 10^{8}$
linear regression coeff $(r)$	0.9912	0.9629	0.9957	0.9720

$$\mathbf{X} = \{^{1}/_{2} + (K_{da}/81[\mathbf{I}]_{0}^{2} + ^{1}/_{4})^{1/2}\}^{1/3} + \{^{1}/_{2} - (K_{da}/81[\mathbf{I}]_{0}^{2} + ^{1}/_{4})^{1/2}\}^{1/3} \quad \text{(for } m = 3) (14)$$

where  $[\mathbf{I}]_0 = [\mathbf{P}_n^*]_{\text{tot}} = [\mathbf{P}_n^*] + m[(\mathbf{P}_n^*)_m]$  and  $K_{\text{da}} = 1/K_a$ . For  $m \ge 4$  the function  $\mathbf{X}$  is to be evaluated only numerically.

Reconstruction of the Experimental Data with the Determined  $k_p$  and  $K_a$ . Finally, we used eqs 12-14 to check whether the  $k_p$  and  $K_a$  values determined in the preceding section allow the reconstruction of the dependence of  $\log r_p$  on  $\log[\mathbf{P}_n^*]_{\text{tot}}$  found experimentally. Plots for  $m \geq 4$  had to be simulated numerically on the basis of eqs 5 and 15. The latter equation stems directly from the expression for the constant of the deaggregation equilibrium (1)  $(K_{da} = [\mathbf{P}_n^*]^m/[(\mathbf{P}_n^*)_m])$ 

$$[\mathbf{P}_{n}^{*}]^{m} + (K_{de}/m)[\mathbf{P}_{n}^{*}] - (K_{de}/m)[\mathbf{I}]_{0} = 0$$
 (15)

as described in ref 18.

In Figure 1 the result of these calculations is shown: the solid and the dashed lines were obtained using values of  $k_p$  and  $K_a$  determined by our method (linear analytical method, eq 8) and the numerical treatment, respectively.

The computed lines reproduce the original dependencies within the calculated error of the experimental work.

#### Conclusions

There are a number of chemical reactions in both lowand high-molecular-weight systems involving species that tend to aggregate. These systems can be described by the following scheme:

$$m\mathbf{P}_n^* \stackrel{K_{da}}{\rightleftharpoons} (\mathbf{P}_n^*)_m \qquad \mathbf{P}_n^*$$
: active

$$\mathbf{P}_n^* + \mathbf{M} \xrightarrow{k_p} \mathbf{P}_{n+1}^* \qquad (\mathbf{P}_n^*)_m$$
: inactive m-aggregate (16)

(a more general notation could have been used).

If no simplifying assumptions are made and if it is only assumed, as shown above, that the aggregated species do not react as such with M, then this system can be solved analytically. Such a solution for the dimerizing active species was briefly outlined by Szwarc,<sup>5</sup> and we have given the full solution in our works,<sup>17,18</sup> allowing the analytical determination of both  $k_p$  and  $K_a$  in the polymerization of  $\epsilon$ -caprolactone initiated with dialkylaluminum alkoxides.

In the present paper the same equation was applied to a few systems known to conform to the kinetic situation described above and treated by the involved authors numerically. These systems are anionic polymerizations of the following monomers: o-methoxystyrene with Li<sup>+</sup> counterion, methyl methacrylate with Li<sup>+</sup> counterion, oxirane with Cs<sup>+</sup> counterion, and hexamethylcyclotrisiloxane (D<sub>3</sub>) with Li<sup>+</sup> counterion.

In all of the analyzed polymerizations a good agreement was found between the values of  $k_{\rm p}$  and  $K_{\rm a}$  determined previously numerically and analytically in this work. The respective data are collected in Table 1.

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